

YEAR 7 — LINES AND ANGLES

Constructing, measuring and using geometric notation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

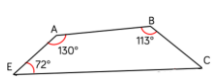
- Use letter and labelling conventions
- Draw and measure line segments and angles
- Identify parallel and perpendicular lines
- Recognise types of triangle
- Recognise types of quadrilateral
- Identify polygons
- Construct triangles (SAS, SSS, ASA)
- Draw Pie charts

Keywords

- Polygon:** A 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two sides the same size
- Right-angled triangle:** a triangle with a right angle
- Frequency:** the number of times a data value occurs
- Sector:** part of a circle made by two radii touching the centre
- Rotation:** turn in a given direction
- Protractor:** equipment used to measure angles
- Compass:** equipment used to draw arcs and circles

Letter and labelling convention

The letter in the middle is the angle
The arc represents the angle

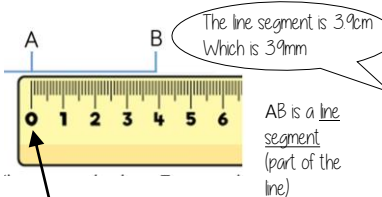


Angle Notation: three letters ABC
This is the angle at B = 113°

Line Notation: two letters EC
The line that joins E to C

Draw and measure line segments

Conversions $1\text{cm} = 10\text{mm}$, $1\text{m} = 100\text{cm}$



Make sure the start of the line is at 0.

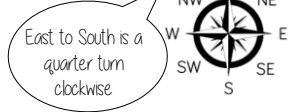
Angles as measures of turn



Clockwise



Anti-Clockwise



East to South is a quarter turn clockwise



Quarter Turn
 90°
Clockwise



Half Turn
 180°

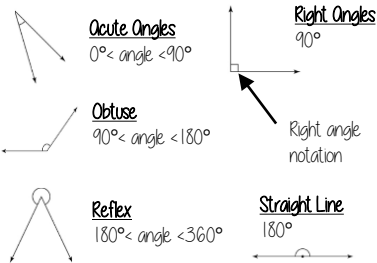


Three-quarter Turn
 270°
Anti-Clockwise

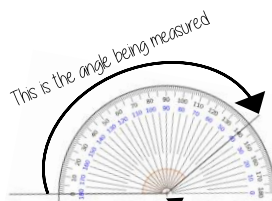


Full Turn
 360°

Classify angles



Measure angles to 180°



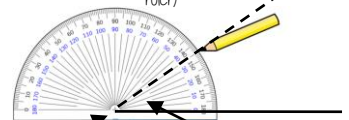
This is the angle being measured
The base line follows the line segment
Make sure the cross is at the point the two lines meet

Read from 0° on the base line
Remember to use estimation
This is an obtuse angle so between 90° and 180°

Draw angles up to 180°

Draw a 35° angle

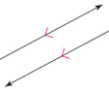
Make a mark at 35° with a pencil
And join to the angle point (use a ruler)



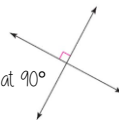
Make sure the cross is at the end of the line (where you want the angle)

Parallel and Perpendicular lines

Parallel lines
Straight lines that never meet
(Have the same gradient)



Perpendicular lines
Straight lines that meet at 90°



Angles over 180°

Use your knowledge of straight lines 180° and angles around a point 360°

360° - smaller angle = reflex angle



Measure the smaller angle first (less than 180°)

Properties of Quadrilaterals

Square
All sides equal size
All angles 90°
Opposite sides are parallel

Rectangle
All angles 90°
Opposite sides are parallel

Rhombus
All sides equal size
Opposite angles are equal



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles

Trapezium
One pair of parallel lines

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Draw Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$

Use a protractor to draw
This is 192°



Polygons

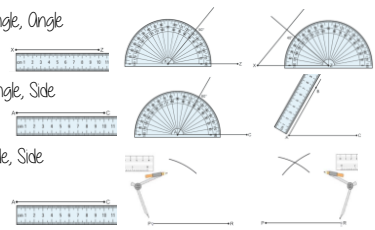
- 3 - Triangle
- 4 - Quadrilateral
- 5 - Pentagon
- 6 - Hexagon
- 7 - Heptagon
- 8 - Octagon
- 9 - Nonagon
- 10 - Decagon

SAS, SSS, ASA constructions

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



If all the sides and angles are the same, it is a **regular** polygon

YEAR 7 — LINES AND ANGLES

Geometric reasoning

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What do I need to be able to do?

By the end of this unit you should be able to:

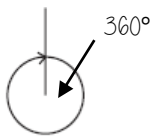
- Understand/use the sum of angles at a point
- Understand/use the sum of angles on a straight line
- Understand/use equality of vertically opposite angles
- Know and apply the sum of angles in a triangle
- Know and apply the sum of angles in a quadrilateral

Keywords

- Vertically Opposite:** angles formed when two or more straight lines cross at a point
- Interior Angles:** angles inside the shape
- Sum:** total, add all the interior angles together
- Convex Quadrilateral:** a four-sided polygon where every interior angle is less than 180°
- Concave Quadrilateral:** a four-sided polygon where one interior angle exceeds 180°
- Polygon:** a 2D shape made with straight lines
- Scalene triangle:** a triangle with all different sides and angles
- Isosceles triangle:** a triangle with two angles the same size and two sides the same size
- Right-angled triangle:** a triangle with a right angle

Sum of angles at a point

The sum of angles around a point is 360°



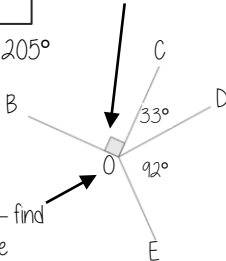
Find angle BOE

$$90^\circ + 33^\circ + 92^\circ = 205^\circ$$

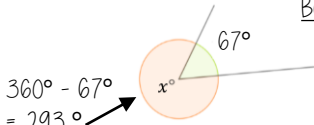
$$360^\circ - 205^\circ$$

$$BOE = 155^\circ$$

Angle notation — 90°

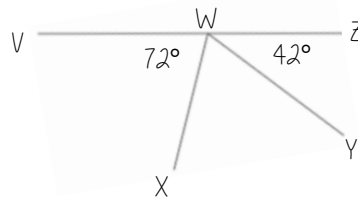


Angle notation — find this missing angle



Sum of angles on a straight line

Adjacent angles that share a common point on a line add up to 180°

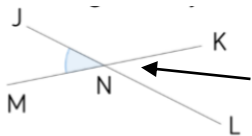


Find angle XWY

$$72^\circ + 42^\circ = 114^\circ$$

$$180^\circ - 114^\circ = 66^\circ$$

Vertically opposite angles

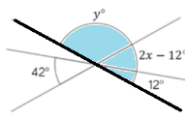


Angle JNM is vertically opposite to angle KNL

$$JNM = KNL$$

Vertically opposite angles are the same

Other angle rules still apply
Look for straight line sums and angles around a point

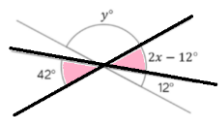


Form equations with information from diagrams

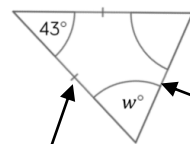
$$2x - 12 = 42$$

$$2x = 54$$

$$x = 27^\circ$$



Sum of angles in triangles



The two base angles will be the same size

Look at triangle notation
This indicates an isosceles triangle

$$\therefore 180 - 43 = 137$$

$$137 \div 2 = 68.5^\circ$$

A triangle can only have ONE right angle

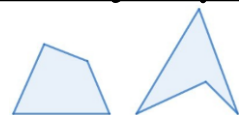
Sum of interior angles in a triangle = 180°



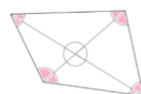
Have a go!
Tearing the corners from triangles forms a straight line which is therefore 180°

Sum of angles in quadrilaterals

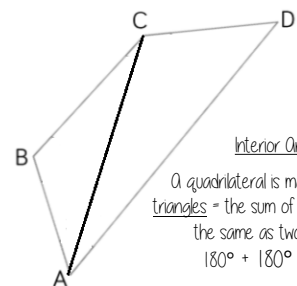
Sum of interior angles in a quadrilateral = 360°



Convex Quadrilateral
Concave Quadrilateral



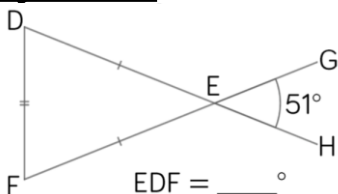
Interior angles are those that make up the perimeter (outline) of the shape



Interior Angles
A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles
 $180^\circ + 180^\circ = 360^\circ$

Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation



EDF = $\underline{\hspace{1cm}}$ $^\circ$

1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH
2. Triangle DEF is isosceles (triangle notation) \therefore EDF = EFD and the sum of interior angles is 180°
 $180^\circ - 51^\circ = 129^\circ$ $129^\circ \div 2 = 64.5^\circ$
3. Angle EDF = 64.5°

Keep working out clear and notes together

YEAR 7 — REASONING WITH NUMBER

Developing number sense

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What do I need to be able to do?

By the end of this unit you should be able to:

- Know and use mental addition/ subtraction
- Know and use mental multiplication/ division
- Know and use mental arithmetic for decimals
- Know and use mental arithmetic for fractions
- Use factors to simplify calculations
- Use estimation to check mental calculations
- Use number facts
- Use algebraic facts

Keywords

Commutative: changing the order of the operations does not change the result

Associative: when you add or multiply you can do so regardless of how the numbers are grouped

Dividend: the number being divided

Divisor: the number we divide by

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Equation: a mathematical statement that two things are equal

Quotient: the result of a division

Mental methods for addition/ subtraction

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction

Mental methods for multiplication/ division

Multiplication is commutative



$$2 \times 4 = 4 \times 2$$

The order of multiplication does not change the result

Partitioning can help multiplication

$$\begin{aligned} 24 \times 6 &= 20 \times 6 + 4 \times 6 \\ &= 120 + 24 \\ &= 144 \end{aligned}$$

Division is not associative

Chunking the division can help $4000 \div 25$
"How many 25's in 100" then how many chunks of that in 4000.

Mental methods for decimals

Multiplying by a decimal < 1 will make the original value smaller e.g. $0.1 = \div 10$

Methods for multiplication 12×0.03

$$\begin{array}{l} 12 \times 3 = 36 \\ 12 \times 3 = 36 \\ 12 \times 0.3 = 3.6 \\ 12 \times 0.03 = 0.36 \end{array} \quad \begin{array}{l} 12 \times 3 = 36 \\ +10 \downarrow +100 \downarrow +1000 \downarrow \\ 12 \times 0.03 = 0.36 \end{array}$$

Methods for division $15 \div 0.05$

Multiply by powers of 10 until the divisor becomes an integer

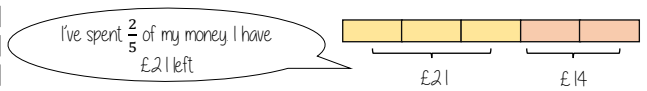
$$\begin{array}{l} 1.5 \div 0.05 \\ \times 100 \downarrow \quad \times 100 \downarrow \\ 150 \div 5 = 30 \end{array}$$

Methods for addition $23 + 24$

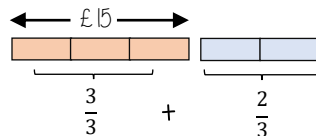
$$\begin{array}{l} 2 + 2 = 4 \\ 0.3 + 0.4 = 0.7 \\ 4 + 0.7 = 4.7 \end{array}$$

Mental methods for fractions

Use bar models where possible



How much did they have to begin with?



What is $\frac{5}{3}$ of £15?

Using factors to simplify calculations

$$30 \times 16$$

$$10 \times 3 \times 4 \times 4$$

$$10 \times 3 \times 2 \times 8$$

$$2 \times 5 \times 3 \times 2 \times 2 \times 2 \times 2$$

$$16 \times 10 \times 3$$

Multiplication is commutative
Factors can be multiplied in any order

Estimation

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

Most estimations round to 1 significant figure

Estimations are useful — especially when using fractions and decimals to check if your solution is possible.

$$210 + 899 < 1200$$

This is true because even if both numbers were rounded up, they would reach $300 + 900$.

The correct estimation would be $200 + 900 = 1100$.

Number facts

Use $124 \times 5 = 620$

For multiplication, each value that is multiplied or divided by powers of 10 needs to happen to the result

$$620 \div 124 = 50$$

For division you must consider the impact of the divisor becoming smaller or bigger.
Smaller — the answer will be bigger (it is being shared into less parts)
Bigger — the answer will be smaller (it is being shared into more parts)

Algebraic facts

$$2a + 2b = 10 \quad \text{Everything } \times 2$$

$$0.1a + 0.1b = 0.5$$

Everything $\div 10$

$$a + b = 5$$

Add 2 to the total

$$a + b + 2 = 7$$

The unknown quantity isn't changing but the variables change what is done to give the result

YEAR 7 — REASONING WITH NUMBER

Sets and probability

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What do I need to be able to do?

By the end of this unit you should be able to:

- Identify and represent sets
- Interpret and create Venn diagrams
- Understand and use the intersection of sets
- Understand and use the union of sets
- Generate sample spaces for single events
- Calculate the probability of a single event
- Understand and use the probability scale

Keywords

Set: collection of things
Element: each item in a set is called an element
Intersection: the overlapping part of a Venn diagram ($A \cap B$)
Union: two ellipses that join ($A \cup B$)
Mutually Exclusive: events that do not occur at the same time
Probability: likelihood of an event happening
Bias: a built-in error that makes all values wrong (unequal) by a certain amount, e.g. a weighted dice
Fair: there is zero bias, and all outcomes have an equal likelihood
Random: something happens by chance and is unable to be predicted

Identify and represent sets

The **universal set** has this symbol ξ — this means **EVERYTHING** in the Venn diagram is in this set

A set is a collection of things — you write sets inside curly brackets { }

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

My sets can include every number between 1 and 50 including those numbers

$A = \{\text{Square numbers}\}$
 $A = \{1, 4, 9, 16, 25, 36, 49\}$

All the numbers in set A are square number and between 1 and 50

Interpret and create Venn diagrams

Mutually exclusive sets
 The two sets have nothing in common
 No overlap

Union of sets
 The two sets have some elements in common — they are placed in the intersection

Subset
 All of set B is also in Set A so the ellipse fits inside the set

The box
 Around the outside of every Venn diagram will be a box. If an element is not part of any set it is placed outside an ellipse but inside the box

Intersection of sets

Elements in the intersection are in set A AND set B

The notation for this is $A \cap B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The element in $A \cap B$ is 15

In this example there is only one number that is both a multiple of 3 and a multiple of 5 between 1 and 15

Union of sets

Elements in the union could be in set A OR set B

The notation for this is $A \cup B$

$\xi = \{\text{the numbers between 1 and 15 inclusive}\}$
 $A = \{\text{Multiples of 5}\}$ $B = \{\text{Multiples of 3}\}$

The elements in $A \cup B$ are 5, 10, 15, 3, 9, 6, 12

There are 7 elements that are either a multiple of 5 OR a multiple of 3 between 1 and 15

This Venn shows the **number of elements** in each set

Sample space — for single events

A sample space represents a possible outcome from an event

They can be interpreted in a variety of ways because they do not tell you the probability

A sample space for rolling a six-sided dice is $S = \{1, 2, 3, 4, 5, 6\}$

A sample space for this spinner is $S = \{\text{Pink, Blue, Yellow}\}$

You only need to write each element once in a sample space diagram

Probability of a single event

Probability = $\frac{\text{number of times event happens}}{\text{total number of possible outcomes}}$

$P(\text{Blue}) = \frac{4}{10}$ ← There are 4 blue sectors
 ← There are 10 sectors overall

Probability notation $P(\text{event}) = \frac{2}{5}$

Probability can be a fraction, decimal or percentage value

$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$

Probability is always a value between 0 and 1

The probability scale

Impossible 0 or 0% Even chance 0.5, $\frac{1}{2}$ or 50% Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Sum of probabilities

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
 ∴ The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$

YEAR 7 — REASONING WITH NUMBER

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Prime numbers and Proof

What do I need to be able to do?

By the end of this unit you should be able to:

- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors
Conjecture: a statement that might be true (based on reasoning) but is not proven
Counterexample: a special type of example that disproves a statement
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3

3, 6, 9, 12, 15...

$3x, 6x, 9x \dots$

This list continues and doesn't end

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Factors

Arrays can help represent factors

Factors of 10: 1, 2, 5, 10

10×1 or 1×10

5×2 or 2×5

The number itself is always a factor

Factors and expressions

$6x \times 1$ OR $6 \times x$

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number

The only even prime number

2

Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Square and triangular numbers

Square numbers

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 ...

odd, even, odd

Triangular numbers

Representations are useful — an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45...

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF — Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers): 1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

LCM — Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Comparing fractions

Compare fractions using a LCM denominator

$\frac{3}{5}$ and $\frac{7}{10}$

$\frac{6}{10}$ and $\frac{7}{10}$

Product of prime factors

Multiplication part-whole models

30 = 2 x 15 = 2 x 3 x 5

30 = 3 x 10 = 3 x 2 x 5

30 = 5 x 6 = 5 x 2 x 3

All three prime factor trees represent the same decomposition

Multiplication is commutative

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

e.g. 60: 30×2 or $2 \times 3 \times 5 \times 2$

150: 30×5 or $2 \times 3 \times 5 \times 5$

Conjectures and counterexamples

Conjecture

1, 2, 4, ...

The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only one counterexample is needed to disprove a conjecture