# YEAR 9 – REASONING WITH GEOMETRY...

# Enlargement & Similarity

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise enlargement and similarity
- Enlarge a shape by a positive SF
- Enlarge a shape from a point
- Enlarge a shape by a fractional SF
- Work out missing sides and angles in a pair of similar shapes.

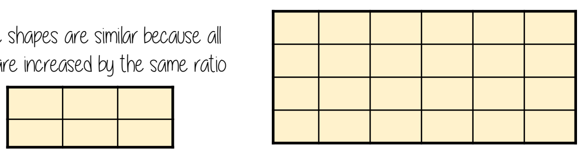
## Keywords

**Similar Shapes**: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.
**Scale Factor**: the multiple describing how much a shape has been enlarged
**Enlarge**: to change the size of a shape (enlargement is not always making a shape bigger)
**Corresponding**: objects (or sides) that appear in the same place in two similar situations.
**Image**: the picture or visual representation of the shape

## Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

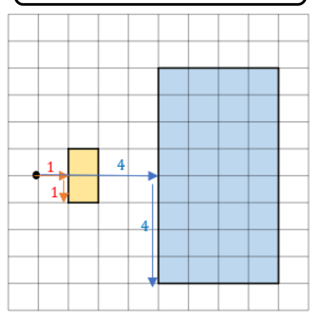
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

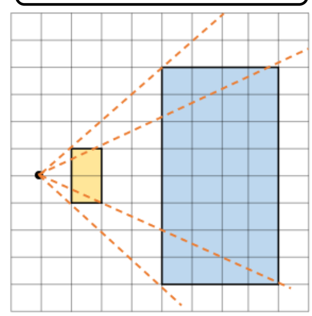
## Enlarge a shape from a point

Scaled distances method




Scale the distance between the point of enlargement and each corresponding vertices

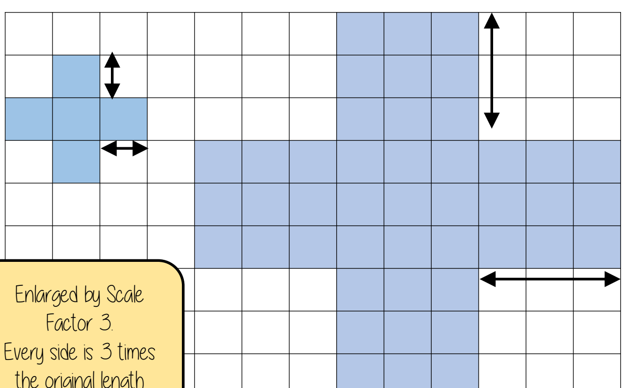
Rays method



Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

## Enlarge by a positive scale factor

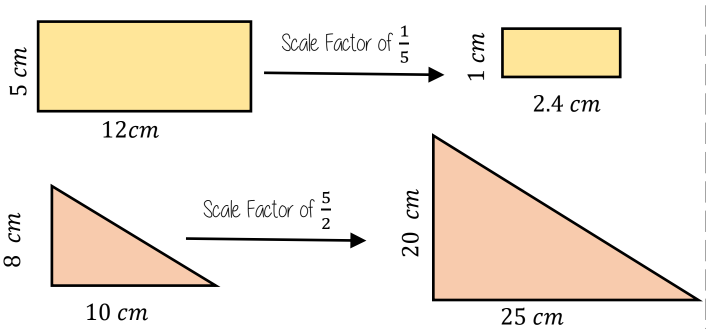
With a scale factor larger than 1 it makes the shape **bigger**



Enlarged by Scale Factor 3.
Every side is 3 times the original length

## Positive fractional scale factor

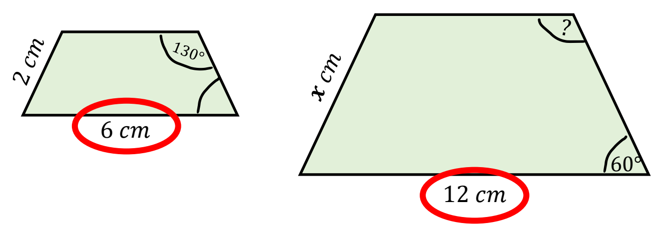
With a scale factor between 0 and 1 it makes the shape **smaller**




## Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle




Corresponding sides identify the scale factor $\frac{12}{6} = 2$ Scale Factor = **2**

Calculate the missing side: Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

Calculate the missing angle: Corresponding angles remain the same

$130°$

# YEAR 9 – REASONING WITH GEOMETRY...

# Solving ratio & proportion problems

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

## Keywords

**Proportion:** a comparison between two numbers

**Ratio:** a ratio shows the relative size of two variables

**Direct proportion:** as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

**Inverse proportion::** as one variable is multiplied by a scale factor the other is divided by the same scale factor.

## Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

This is a multiplicative change

4 cans of pop = £2.40 (× 0.5)
2 cans of pop = £1.20 (× 0.5)

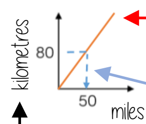
This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40 (× 3)
12 cans of pop = £7.20 (× 3)

Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

## Conversion Graphs

Compare two variables




This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

## Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

| | | | |
|---|---|---|---|
| T | 1 | 2 | 8 |
| G | 40 | 20 | 5 |

× 2 ÷ 4

## Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

**Shop A**
4 cans for £1.20
£1.20 ÷ 4
1 can is £0.30
Or 30p

**Shop B**
3 cans for 93p
£0.93 ÷ 3
1 can is £0.31
Or 31p

Cost per item

Shop A is the best value as it is 1p cheaper per can of pop

**Shop A**
4 cans for £1.20
4 ÷ £1.20
£1 buys 3.333 cans of pop

3 cans for 93p
3 ÷ £0.93
£1 buys 3.23 cans of pop

Cost per pound

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

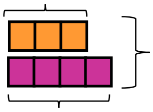
## Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4

James



£350

Lucy

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

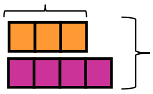
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James: Lucy
3 : 4 (× 50)
£150:£200

James = 3 x £50 = £150



£350

Lucy = 4 x £50 = £200

## Finding a value given 1:n (or n:1)

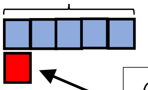
Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

□ = one part = 10 pens

Blue pens



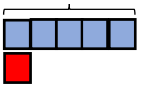
Red pens

One unit = 10 pens

Put back into the question

Blue : Red
5 : 1 (x 10)
50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

# YEAR 9 – REASONING WITH GEOMETRY… Rates

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

## Keywords

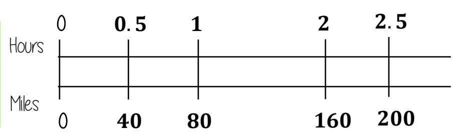
**Convert**: change
**Mass**: a measure of how much matter is in an object. Commonly measured by weight.
**Origin**: the coordinate (0, 0)
**Volume**: the amount of 3D space a shape takes up
**Substitute**: putting numbers where letters are – replacing numbers into a formula

## Speed, Distance, Time

"per" for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

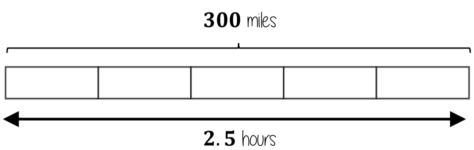
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



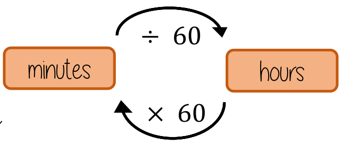

e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.

Bar models can help to calculate mph




Each part is half an hour
Each part is 60 miles

## Speed, Distance, Time




**Before** calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

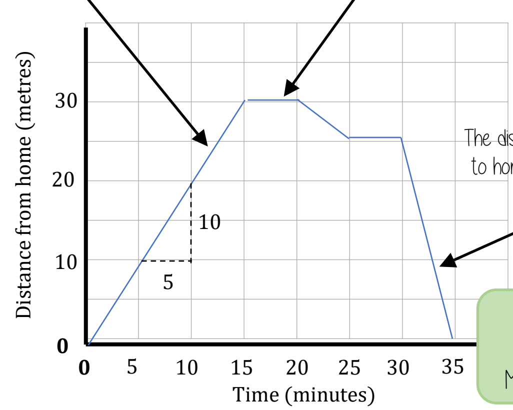
## Distance – Time graphs

The steeper a gradient the faster the speed

Gradient = speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still




The distance coming closer to home shows the return journey

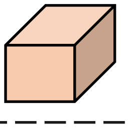
Units are important. Meters per minute

## Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

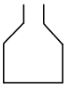
$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$

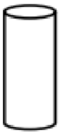


volume of prism = Area of cross section × Depth

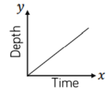
## Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



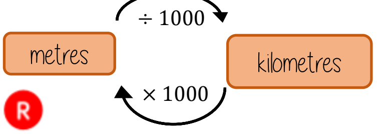

Units are important. Ensure any volume calculations are the same unit as the rate of flow

## Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

**Speed**: miles **per** hour
**Exchange rates**: euros **per** pounds
**Density**: mass **per** volume



# YEAR 9 – REPRESENTATIONS...

# Probability

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## What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

## Keywords

**Probability**: the chance that something will happen
**Relative Frequency**: how often something happens divided by the outcomes
**Independent**: an event that is not effected by any other events.
**Chance**: the likelihood of a particular outcome.
**Event**: the outcome of a probability – a set of possible outcomes.
**Biased**: a built in error that makes all values wrong by a certain amount.

## The probability scale

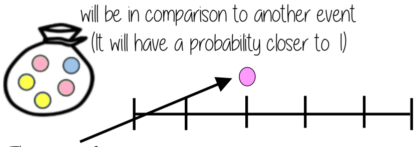
R

Impossible 0 or 0%

Even chance 0.5, $\frac{1}{2}$ or 50%

Certain 1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

## Single event probability

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

| Dark | Milk | White |
|---|---|---|
| 0.15 | 0.35 | |

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

## Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

| Colour | Frequency | Relative Frequency |
|---|---|---|
| Green | 6 | 0.3 |
| Yellow | 12 | 0.6 |
| Blue | 2 | 0.1 |
| | 20 | |

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$0.3 \times 100 = 30$

## Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

| Dark | Milk | White |
|---|---|---|
| 0.15 | 0.35 | 0.5 |

The sum of the probabilities is 1

On experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$0.15 \times 400 = 60$

## Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

**Probability of event 1 × Probability of event 2**

Find the probability of getting a 5 and a red

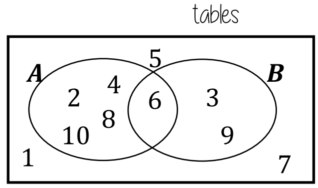
$P(5) = \frac{1}{6}$ $\quad P(R) = \frac{1}{4}$

$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

## Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables

R




| | Car | Bus | Walk | Total |
|---|---|---|---|---|
| Boys | 15 | 24 | 14 | 53 |
| Girls | 6 | 20 | 21 | 47 |
| Total | 21 | 44 | 35 | 100 |

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| H | 1,H | 2,H | 3,H | 4,H | 5,H | 6,H |
| T | 1,T | 2,T | 3,T | 4,T | 5,T | 6,T |

# YEAR 9 – REPRESENTATIONS…

# Algebraic Representation

@whisto_maths

## What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

## Keywords

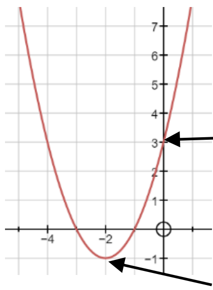
**Quadratic**: a curved graph with the highest power being 2. Square power.
**Inequality**: makes a non equal comparison between two numbers
**Reciprocal**: a reciprocal is $1$ divided by the number
**Cubic**: a curved graph with the highest power being 3. Cubic power.
**Origin**: the coordinate (0, 0)
**Parabola**: a 'u' shaped curve that has mirror symmetry

## Quadratic Graphs

$y = x^2 + 4x + 3$

If $x^2$ is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape




Intersection with the $y$ axis

Substitute the $x$ values into the equation of your line to find the $y$ coordinates

| $x$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|---|
| $y$ | $3$ | $0$ | $-1$ | $0$ | $3$ | $8$ |

Coordinate pairs for plotting $(-3, 0)$

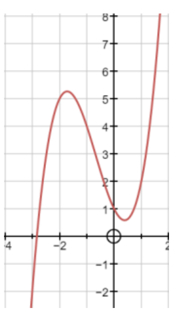
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

## Interpret other graphs

### Cubic Graphs

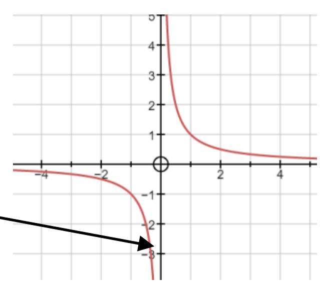
$y = x^3 + 2x^2 - 2x + 1$




If $x^3$ is the highest power in your equation then you have a cubic graph

### Reciprocal Graphs

$y = \frac{1}{x}$

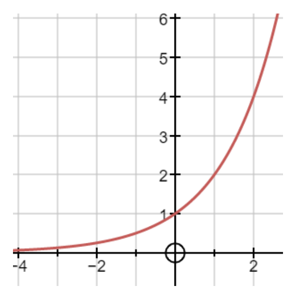



Reciprocal graphs never touch the $y$ axis.
This is because $x$ cannot be $0$
This is an asymptote

### Exponential Graphs

$y = 2^x$

Exponential graphs have a power of $x$

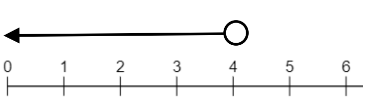


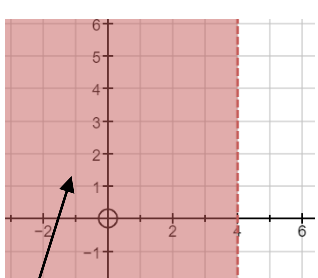

## Represent Inequalities

Multiple methods of representing inequalities

$x < 4$

All values are less than $4$



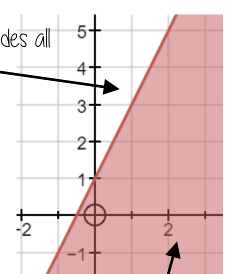




The shaded area indicates all possible values of $x$

The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$y \geq 2x + 1$




The shaded area indicates all possible solutions to this inequality