YEAR 10 – GEOMETRY...

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Angles and bearings

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

Keywords

Cardinal directions: the directions of North, South, East, West

Angle: the amount of turn between two lines around their common point

Bearing: the angle in degrees measured clockwise from North.

Perpendicular: where two lines meet at 90°

Parallel: straight lines always the same distance apart and never touch. They have the same gradient.

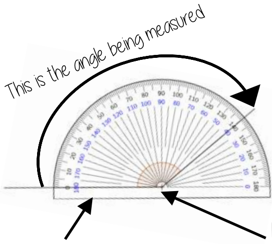
Clockwise: moving in the direction of the hands on a clock.

Construct: to draw accurately using a compass, protractor and or ruler or straight edge.

Scale: the ratio of the length of a drawing to the length of the real thing.

Protractor: an instrument used in measuring or drawing angles.

Measure angles to 180°



Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90 ° and 180 °

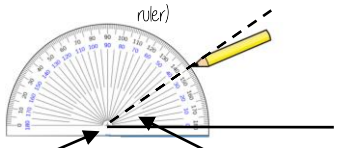
The base line follows the line segment

Make sure the cross is at the point the two lines meet

Draw angles up to 180°

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)



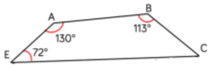
Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation

The letter in the middle is the angle

The arc represents the part of the angle



Angle Notation: three letters $\boldsymbol{A\hat{B}C}$ This is the angle at B = 113 °

$\angle \boldsymbol{ABC}$ is also used to represent the angle at B.

Scale drawings

1 : 20

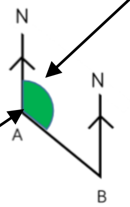
For every 1cm on the model there are 20cm in real life

Remember: Scale drawings ONLY change lengths and distances. Angles remain the same

Understand and represent bearings

- A bearing is always measured from NORTH
- It is always given as three figures

The angle indicated starts from the North line at A and joins the path connecting A to B.



This angle shows the bearing of **B from A**

The bearing of B from A is calculated by measuring the highlighted angle

The sentence… "Bearing of ____ from _____" is really important in identifying the bearing being represented

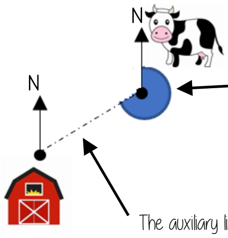
Using <u>estimation</u> it is clear this angle is between 090° and 180°

Directions

Clockwise

Anti-Clockwise

Measure and read bearings



The bearing of the cow to the barn

This angle is measured from NORTH

It is measured in a clockwise direction

<u>Estimation</u> indicates this angle is between 180° and 270°

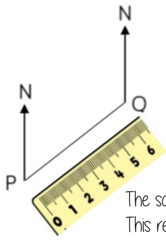
Use a protractor to measure accurately

Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember – angles DO NOT change size in scaled drawings



The bearing measurements do not change from "real life" to images

The units in the ratio scale are the same

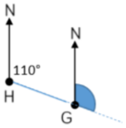
6cm = 30km

6:3,000,000

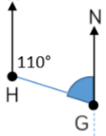
The scale may need to be calculated from the image. This represents 30km from P to Q.

Bearings with angle rules

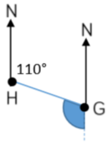
Because two North lines are PARALLEL….



They form <u>corresponding angles</u> and therefore are the same size



They form <u>co-interior angles</u> and add up to 180°

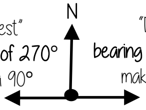


They form <u>alternate angles</u> and therefore are the same size

Bearings with right-angled geometry

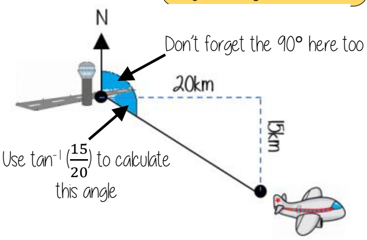
Look for Right-angles:
Pythagoras
Trigonometry (Sin, Cos, Tan)

"Due West"
bearing of 270°
makes a 90° angle



"Due East"
bearing of 090°
makes a 90° angle

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off.



Don't forget the 90° here too

Use $\tan^{-1}\left(\frac{15}{20}\right)$ to calculate this angle

YEAR 10 – GEOMETRY...

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Working with circles

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

Keywords

Circumference: the length around the outside of the circle – the perimeter

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

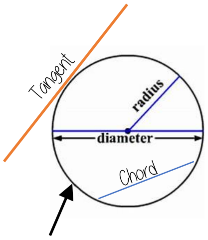
Chord: a line segment connecting two points on the curve

Frustum: a pyramid or cone with the top cut off

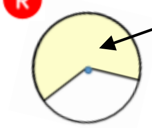
Hemisphere: half a sphere

Surface area: the total area of the surface of a 3D shape.

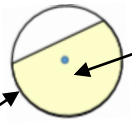
Parts of a circle



Circumference



Sector (part of the circle made from two radii)



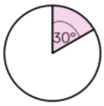
Segment (part of the circle made from a chord)

An arc is a part of the circumference

Fractional parts of a circle

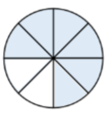
A circle is made up of 360°

Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



30° represents $\frac{30}{360}$ of a full circle

$\frac{30}{360} = \frac{1}{12}$



$\frac{270}{360}$ of a full circle (in degrees)

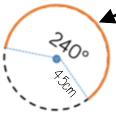
$\frac{6}{8}$ of a full circle (in equal parts)

$\frac{3}{4}$ of a full circle

The fraction of the circle is as $\frac{\theta}{360}$

θ represents the degrees in the sector

Arc length



Remember an arc is part of the circumference

Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$

Arc length = $\frac{\theta}{360}$ ×circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi$ $\underline{= 6\pi}$

Perimeter

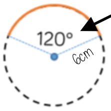
Perimeter is the length around the outside of the shape

This includes the arc length and the radii that encloses the shape

Perimeter = $\frac{\theta}{360}$ ×circumference $+ 2r$

$\underline{= 6\pi + 9}$

Sector area



Remember a sector is part of a circle

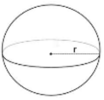
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$

Sector area = $\frac{\theta}{360}$ × area of circle

$= \frac{120}{360} \times 36\pi$

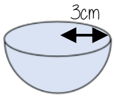
$= \frac{1}{3} \times 36\pi$ $\underline{= 12\pi}$

Volume of a sphere



Volume Sphere = $\frac{4}{3}\pi r^3$

NOTE: This is now a **cubed** value



Volume Sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3$

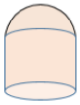
$= \frac{4}{3} \times \pi \times 27$ $\underline{= 36\pi}$

A hemisphere is half the volume of the overall sphere

$= 36\pi \div 2$

$= 18\pi$

Look out for hemispheres being placed on other 3D shapes, e.g cones and cylinders



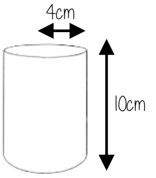
Volume of a cone and a cylinder

Volume Cylinder= $\pi r^2 h$

A cylinder is a prism – cross section is a circle

Volume Cone = $\frac{1}{3}\pi r^2 h$

A cone is a pyramid with a circular base



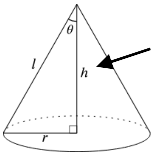
$V = \pi r^2 h$

$= \pi \times 4^2 \times 10$

$= \pi \times 160$

$\underline{= 160\pi}$ cm²

Give your answer in terms of π' means NOT in terms of pi $\underline{= 502.7}$cm²

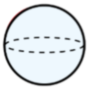


The height of a cone is the perpendicular height from the vertex to the base

Look out for trigonometry or Pythagoras theorem – the radius forms the base of a right-angled triangle

Surface area of a sphere

Surface area = $4\pi r^2$



Radius = 5cm

Surface area = $4\pi r^2$

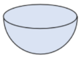
$= 4 \times \pi \times 5^2$

$= 4 \times \pi \times 25$

$= \underline{100\pi}$

The curved surface area of a sphere

A hemisphere has the curved surface AND a flat circular face



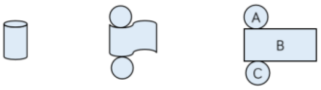
$= 100\pi \div 2 = 50\pi$

$= 50\pi + \pi \times 5^2$

Hemisphere $\underline{= 75\pi}$

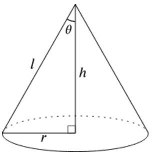
Surface area of cones and cylinders

Surface area cylinder= $2\pi r^2 + \pi dh$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



Curved surface area Cone = $\pi r l$

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

YEAR 10 – GEOMETRY...

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Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Scalar: a single number used to represent the multiplier when working with vectors

Column vector: a matrix of one column describing the movement from a point

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

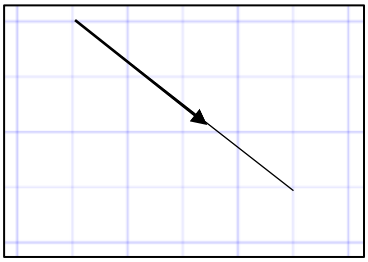
Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis →

Movement along the y-axis ↗

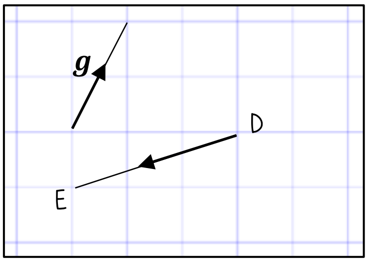
$\begin{pmatrix}4\\-3\end{pmatrix}$ →



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$

Addition of vectors

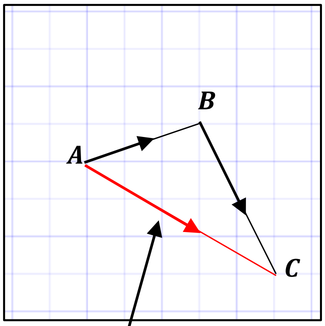
$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$

$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$$

$$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$



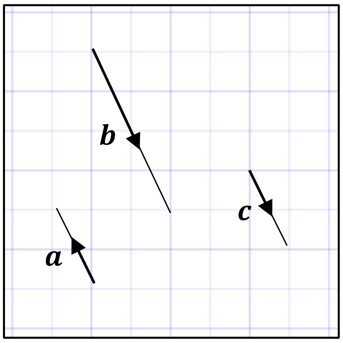
The resultant

Look how this addition compares to the vector $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{a} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix}1\\-2\end{pmatrix}$

$$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$$

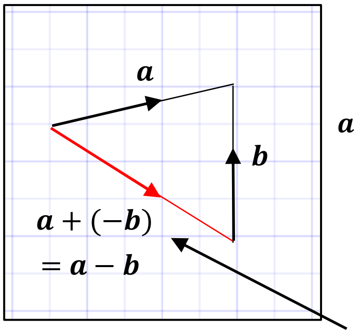
Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$$

Addition and subtraction of vectors



$\boldsymbol{a} = \begin{pmatrix}5\\1\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}0\\4\end{pmatrix}$

$$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix}5+ & -0\\1+ & -4\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix}$$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

YEAR 10 – PROPORTION...

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Ratios and fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare

Equivalent: of equal value

Proportion: a statement that links two ratios

Integer: whole number, can be positive, negative or zero.

Fraction: represents how many parts of a whole.

Denominator: the number below the line on a fraction. The number represent the total number of parts..

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Origin: (0,0) on a graph. The point the two axes cross

Gradient: The steepness of a line

Compare with ratio R

"For every dog there are 2 cats"

The ratio has to be written in the same order as the information is given.

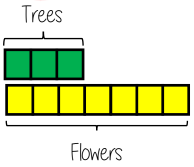
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction R

Trees: Flowers

3 : 7



Ratio

Fraction of trees

$$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} \quad \frac{3}{10}$$

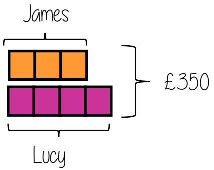
Fraction

Sharing a whole into a given ratio R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

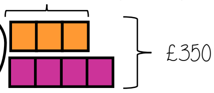
£350 ÷ 7 = £50

☐ = one part
= £50

Put back into the question

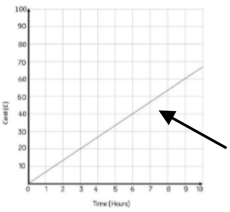
James: Lucy
× 50 3 : 4 × 50
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Ratio and graphs R



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale R

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
× 10 1cm : 30cm × 10
10cm : 300cm

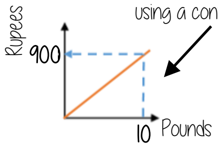
Conversion between currencies R

£1 = 90 Rupees ← Currency is directly proportional

For every £1
I have 90 Rupees

× 10 £1 = 90 Rupees × 10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

× 7 £1 = 90 Rupees × 7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

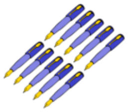
The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Best buys



You could work out how much 40 pens are and then compare

	4 pens costs £2.60	10 pens costs £6.00
"1 pen costs..."	£2.60 ÷ 4 = **£0.65**	£6.00 ÷ 10 = **£0.60**
"1-pound buys..."	4 ÷ 2.60 = **1.54 pens**	10 ÷ 6 = **1.67 pens**

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

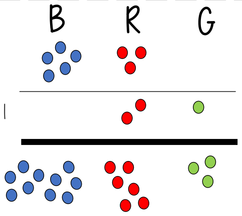
The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10 — PROPORTION...

Percentages and Interest

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

Keywords

Exponent: how many times we use a number in multiplication. It is written as a power

Compound interest: calculating interest on both the amount plus previous interest

Depreciation: a decrease in the value of something over time.

Growth: where a value increases in proportion to its current value such as doubling.

Decay: the process of reducing an amount by a consistent percentage rate over time.

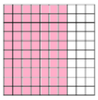
Multiplier: the number you are multiplying by

Equivalent: of equal value.

Compare FDP

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator

$\frac{\blacksquare}{\square}$ → S⇔D Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals

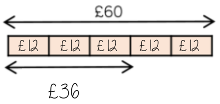
eg $\frac{1}{3}$ = 0.3333333

= $0.\dot{3}$

The dot above the 3

Fraction/ Percentage of amount

Find $\frac{3}{5}$ of £60



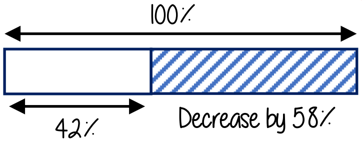
Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6

60% of £60
= 0.6 x 60
= £36

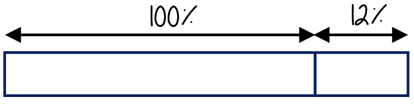
Percentage increase/decrease



Decrease by 58%

100% - 58% = 42%

1.00 – 0.58 = 0.42 ← **Multiplier** Less than 1

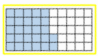


Increase by 12%

100% + 12% = 112%

1.00 + 0.12 = 1.12 ← **Multiplier** More than 1

Express as a percentage



27 per every 50 shaded $\frac{27}{50}$ 54 per every 100 shaded $\frac{54}{100}$ 54%

$\frac{13}{30}$ → $\frac{\blacksquare}{\square}$ → $\frac{13}{30}$ → × 100

43.3333...%

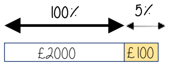
→ 43%

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage

Simple and compound interest

Simple Interest

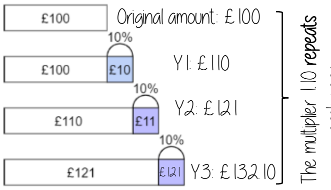


James invests £2000 at 5% simple interest

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years



Original amount: £100

Y1: £110

Y2: £121

Y3: £132.10

The multiplier 1.10 **repeats** each year

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

$$£100 \times 1.10^{3}$$

Original amount

Repeated multiplier

Number of occurrences

Depreciation

Depreciation calculations use multipliers **less than** 1

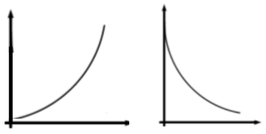
Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

e.g. Increase of 10% then a reduction of 10%

$\underbrace{\times 1.10 \quad \times 0.9}$

x 0.99 The multiplier

Growth and decay



Compound growth Compound decay

Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

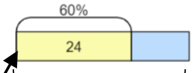
Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test

Original x 0.6 = 24

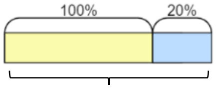


24 ÷ 0.6 = 40 marks

10% = 6
100% = 40

Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?



£3000

Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

YEAR 10 — PROPORTION...

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Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

Keywords

Event: one or more outcomes from an experiment

Outcome: the result of an experiment.

Intersection: elements (parts) that are common to both sets

Union: the combination of elements in two sets.

Expected Value: the value/ outcome that a prediction would suggest you will get

Universal Set: the set that has all the elements

Systematic: ordering values or outcomes with a strategy and sequence

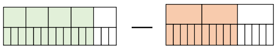
Product: the answer when two or more values are multiplied together.

Add, Subtract and multiply fractions

R

Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$



$$\underbrace{\frac{12}{15} - \frac{10}{15}} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

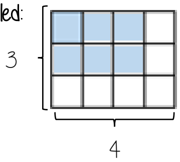
$$\frac{3}{4} \times \frac{2}{3}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Parts shaded

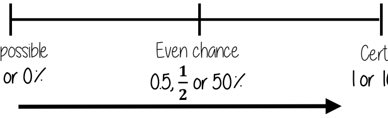
Total number of parts in the diagram

Modelled:



Likeliness of a probability

R



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

Sum to 1

R

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability	What we expect to happen
Experimental probability	What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Sample space

The possible outcomes from rolling a dice

R

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tales) = $\frac{3}{12}$

Independent events

The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

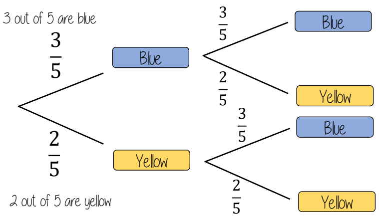
$$P\ (A \text{ and } B)$$
$$= P\ (A) \times P\ (B)$$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability

3 out of 5 are blue



$= P\ (BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P\ (BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$= P\ (YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

$= P\ (YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

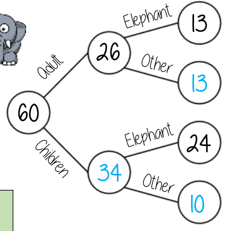
Sum of probabilities = 1

2 out of 5 are yellow

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday mornin 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

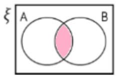
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Two-way table

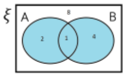
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



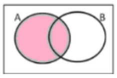
in set A AND set B

$P\ (A \cap B)$



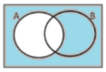
in set A OR set B

$P\ (A \cup B)$



in set A

$P\ (A)$



NOT in set A

$P\ (A')$

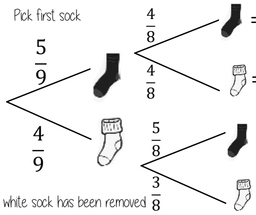
Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks., Jamie picks 2 socks from the drawer.

Pick first sock



$= P\ (BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P\ (BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P\ (WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P\ (WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

A white sock has been removed

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick.