Year 11 – Graphs…

Gradients and lines

What do I need to be able to do?

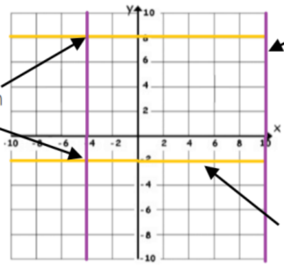
- Equations of lines parallel to the axis
- Plot and interpret $y = mx + c$
- Find the equation of a straight line from a graph, given one point and gradient, or from two points

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis
Parallel: two lines that never meet with the same gradient
Co-ordinate: a set of values that show an exact position on a graph given as (x, y)
Linear: linear graphs (straight line) – linear difference by addition/subtraction
Mid-point: The middle of. The point halfway along.

Lines parallel to the axis

R



Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Interpreting $y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

The equation of a line can be rearranged. Eg:

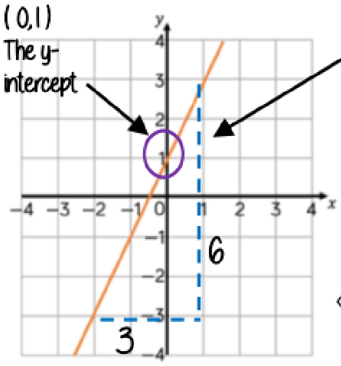
y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph

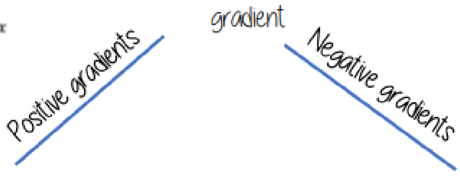
(0,1)
The y-intercept



The Gradient
$\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient



Equation of line – given point and gradient

A line has a gradient of -2 and passes through the point $(1, -4)$

What is the equation of the line?

$$y = mx + c$$

We know the gradient is -2 so…

$$y = -2x + c$$

We now know that the point (1, -4) is on the line so we can substitute these values in as $x = 1$ and $y = -4$ and then solve to get c:

$$-4 = -2(1) + c$$
$$-4 = -2 + c$$
$$-4 + 2 = c$$
$$c = -2$$

Therefore the equation of the line is:

$$y = -2x - 2$$

Plotting $y = mx + c$ graphs

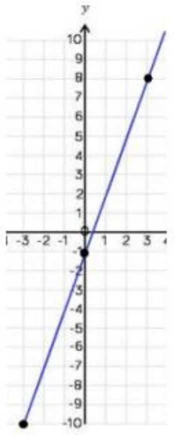
R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	–3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)

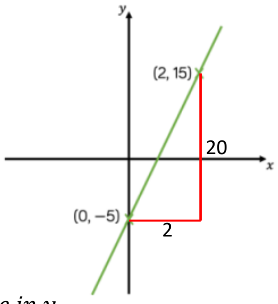


You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Finding the equation of the line from two points



The y-intercept is at -5

To calculate the gradient (m) from two points on a graph: $m = \frac{change\ in\ y}{change\ in\ x}$

$$m = \frac{20}{2}$$
$$m = 10$$

Therefore the equation of the graph is:

$$y = 10x - 5$$

Finding the gradient from two points

Gradient of line between $(2, 5)$ and $(3, 14)$

$$m = \frac{change\ in\ y}{change\ in\ x}$$

$$m = \frac{14 - 5}{3 - 2}$$
$$m = \frac{9}{1}$$

Therefore the gradient between the two points is 9

Year 11 – Graphs…
Non – linear graphs

What do I need to be able to do?

- Plot and read quadratic graphs
- Plot and read cubic graphs
- Plot and read from reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercepts of quadratics

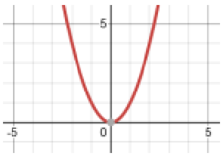
Keywords

Root: Solutions to graph. Where the graph crosses x axis
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis
Parallel: two lines that never meet with the same gradient
Co-ordinate: a set of values that show an exact position on a graph
Quadratic: x^2 the highest exponent of the variable (usually x) is a square
Cubic: x^3 the highest exponent of the variable is three
Reciprocal: a pair of numbers that multiply together to give 1

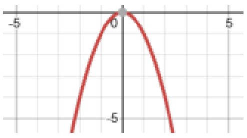
Quadratic graphs

A quadratic graph will always be in the shape of a parabola.

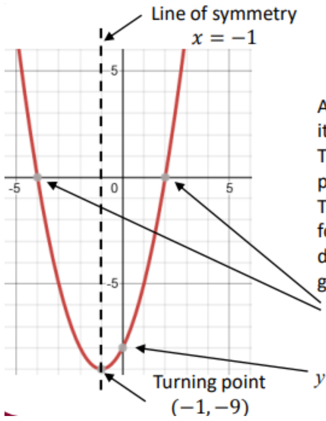
$y = x^2$



$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.



Examples

$y = x^2 + 2x - 8$

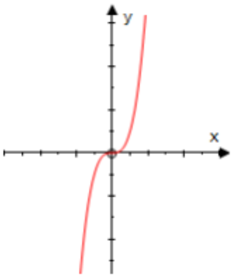
A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation. There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept = -8

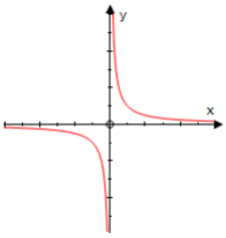
Cubic graphs

- The highest power of x is 3.
- The graph to the right shows $y = x^3$
- Other examples of cubic graphs could be $y = x^3 - 5$, $y = 2x^3$ or $y = x^3 + x^2 + 1$



Reciprocal graphs

- The graph shows $y = \frac{1}{x}$
- In this example, there are asymptotes at the x and y axes – this means that the graph gets closer and closer to the axes without ever touching them.



Completing table of values

If you have to draw your own table of values use x values from -2 to 2

Complete the table for $y = x^2 + 2x - 4$

x	-4	-3	-2	-1	0	1	2
y	4					-1	

Substitute x values into the equation you are given to find the values of y

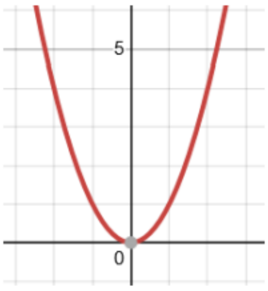
When $x = -4$ $\quad y = -4^2 + (2 \times -4) - 4$
$y = 16 + -8 - 4$
$y = 4$

When $x = 1$ $\quad y = 1^2 + (2 \times 1) - 4$
$y = 1 + 2 - 4$
$y = -1$

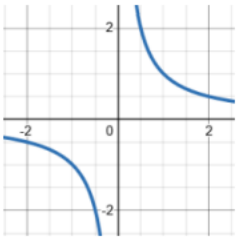
This is a coordinate of $(1, -1)$ which can be plotted to create a graph

Types of graphs

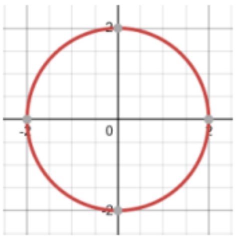
You need to be able to recognise different types of graphs



Quadratic graphs
$y = x^2$



Reciprocal graphs
$y = \frac{1}{x}$



Circle graphs
$x^2 + y^2 = 4$

Cubic graphs
$y = x^3$

Year 11 – Graphs… using graphs

What do I need to be able to do?

- Construct and interpret conversion graphs and other real-life straight line graphs
- Construct and interpret distance/time and speed/time graphs
- Recognise and interpret proportion graphs

Keywords

Convert: Change a value or expression from one form to another
Gradient: How steep a line is
Intercept: Where two graphs cross
Distance-time graph: A graph that shows a journey and the relationship between the distance reached in a given time
Speed-time graph: A graph that shows the relationship between the speed and time taken
Real-life graph: This is a graph that represents a situation that we would see in real life
Direct Proportion: A relationship between two quantities such that as one increases, the other increases (or as one decreases, the other decreases) at the same rate.
Inverse proportion: A relationship between two quantities such that as one increase, the other decrease

Conversion graphs

Change £80 into Turkish lira

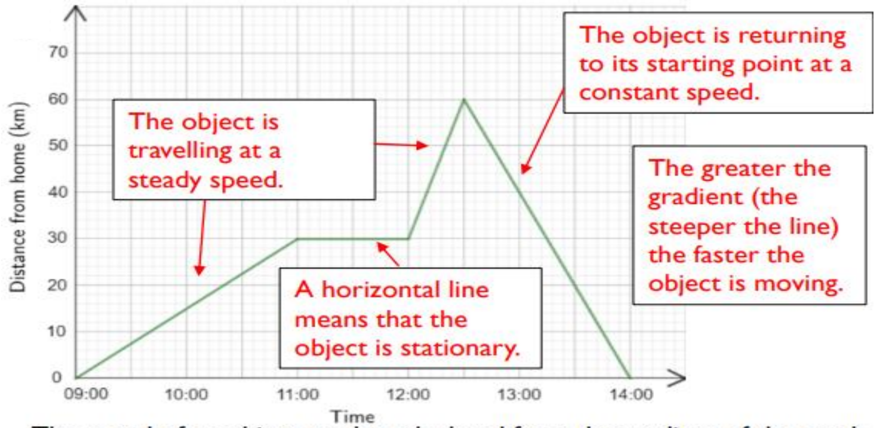
1) Start at 80 on the horizontal axes as this for pounds and go up vertically until you reach the line
2) From the line, read horizontally until you get to the axis showing lira

Change 600 Turkish lira to pounds

As this value is not shown by the graph, we have to use a value that is to help.

1) Start at 200 on the vertical axes and go across horizontally until you reach the line. From the line, read vertically until you get to the axes.
2) 200 lira = £50
   600 lira = £150

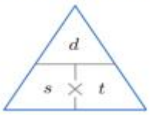
Distance-time graph



The speed of an object can be calculated from the gradient of the graph.

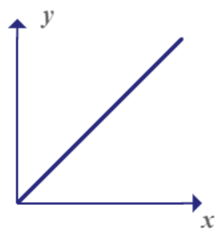
E.g. calculate the speed at which the object travelled between 9am and 11am.

Speed = 30 ÷ 2
= 15 km/hr

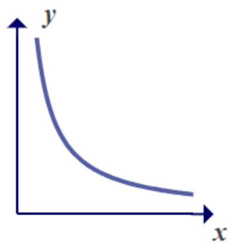


Proportion graphs

Direct proportion graph



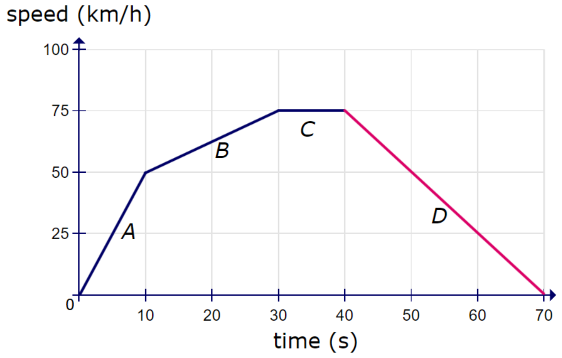
Inverse proportion graph



Speed-time graph

Here is a speed time graph showing the speed a motorbike is travelling at as time goes on.

A- the motorbike is **accelerating** quite hard
B – the bike is still **accelerating**, but less hard
C – The bike is now travelling at a **constant speed** of 75km/hr
D – The bike is **slowing down** at a constant rate



Year 11 – Algebra…
Expanding and factorising

What do I need to be able to do?

- Expand and simplify brackets
- Factorise with single brackets
- Factorise quadratic expressions
- Solve equations equal to 0
- Solve quadratic equations by factorisation

Keywords

Expand: multiply each term in the bracket by expression outside the bracket
Simplify: collect like terms
Factorise: reverse of expanding. Taking out a common factor
Quadratic: The highest power of the variable is squared e.g. x^2
Expression: numbers, symbols and operators grouped together to show the value of something, no equals sign
Equation: shows that two expressions are equal, it will have an equals sign =
Solve: find a numerical value that satisfies an equation

Expand and simplify with a single bracket

R

Expand means 'multiply out'

$5(3x + 4)$
$= 5 \times 3x + 5 \times 4$
$= 15x + 20$

Expand each of the brackets first and then simplify

$3(2x - 1) - 4(3x - 2)$
$= 6x - 3 - 12x + 8$
$= -4x + 5$

You could also use a grid to expand the brackets

×	3x	+4
5	15x	+20

$= 15x + 20$

Factorise into a single bracket

R

Factorise $12x^2yz - 27xz$
$3xz \times 4xy - 3xz \times 9$
$3xz(4xy - 9)$

Factorise means put back into brackets by taking out a common factor

Factorise quadratic expressions

Factorise $x^2 + x - 42$

Sum of (+) 1
$-6 + 7 = 1$

product of -42
$-6 \times 7 = -42$

$x^2 + x - 42 = (x - 6)(x + 7)$

Expand binomials

Use the grid method to expand brackets

$(5x - 3)(2x + 1)$

×	5x	−3
2x	$10x^2$	$-6x$
+1	+5x	−3

$= 10x^2 - 6x + 5x - 3$
$= 10x^2 - x - 3$

Remember to simplify the $-6x$ and $+5x$

Another method

$(3x + 2)(5x + 3)$
$= 15x^2 + 9x + 10x + 6$
$= 15x^2 + 19x + 6$

Don't forget to simplify $9x + 10x = 19x$

Solve quadratic equations by factorisation

The highest common factor of $9x^2$ and $27x$ is $9x$

Solve $9x^2 - 27x = 0$
$9x(x - 3) = 0$
$9x = 0 \quad x - 3 = 0$
$x = 0 \quad x = 3$

Solve $x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x + 5 = 0 \quad x - 4 = 0$
$x = -5 \quad x = 4$

We need 2 numbers with a sum of +1 and a product of -20
$+5$ and -4

Before you can factorise and solve, the equation must be in the form $x^2 + bx + c = 0$

Solve $x^2 = 2x + 15$
Rearrange
$x^2 - 2x - 15 = 0$
$(x - 5)(x + 2) = 0$
$x - 5 = 0 \quad x + 2 = 0$
$x = 5 \quad x = -2$

Solve equations equal to zero

One or both of terms would have to equal 0

$3x + 4 = 0$
$-4 \quad -4$
$3x = -4$
$\div 3 \quad \div 3$
$x = \frac{-4}{3}$

$9x(x - 3) = 0$
$9x = 0 \quad x - 3 = 0$
$x = 0 \quad x = 3$

$(2x + 1)(1 - x) = 0$

$2x + 1 = 0$
$-1 \quad -1$
$2x = -1$
$\div 2 \quad \div 2$
$x = \frac{-1}{2}$

$1 - x = 0$
$+x \quad +x$
$x = 1$

Year 11 – Algebra…
Changing the subject

What do I need to be able to do?

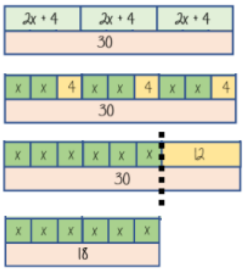
- Solve linear equations
- Solve inequalities
- Form and solve equations and inequalities in context of shape
- Change the subject formula (Simple/known/complex)

Keywords

Equation: shows that two expressions are equal, it will have an equals sign =
Solve: find a numerical value that satisfies an equation
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another
Change: Rearrange the equation
Rearrange: Change the order
Inverse operation: the operation that reverses the action

Solve equations

R



3 (2x + 4) = 30

3 (2x + 4) = 30 — Expand the brackets

6x + 12 = 30
−12 −12

6x = 18
÷ 6 ÷ 6

x = 3

Substitute to check your answer. This could be negative or a fraction or decimal

Language of rearranging…

Make XXX the subject
Change the subject
Rearrange…

There are many ways in which a rearranging question can be asked so look out for these keywords

Solve inequalities

R

Solving inequalities has the same method as equations

5 (x + 4) < 3 (x + 2)
5x + 20 < 3x + 6
2x + 20 < 6
2x < − 14
x < − 7

Check it!

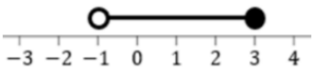
5 (−8 + 4) < 3 (−8 + 2)
5 (−4) < 3 (−6)
−20 < −18
✓ −20 IS smaller than − 18

Showing inequality solutions on a number line

$x < 1$ $x \leq 1$ $x > 1$ $x \geq 1$

Both represent values less than 1
Includes the value 1
Both represent values more than 1
Includes the value 1

Values less than or equal to 3 but also more than − 1

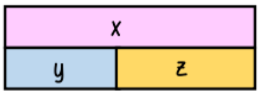


$-1 < x \leq 3$

This includes the integer values 0, 1, 2, 3

● Includes the value it sits above
○ Does NOT include the value it sits above

Change the subject of formula



x = y + z
Rearrange to make y the subject
y = x − z

y ⟶ +z ⟶ x
y ⟵ −z ⟵ x

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution

In an equation (find x)

4x − 3 = 9
+3 +3
4x = 12
÷ 4 ÷ 4
x = 3

In a formula (make x the subject)

xy − s = a
+ s + s
xy = a + s
÷ y ÷ y
$x = \frac{a + s}{y}$

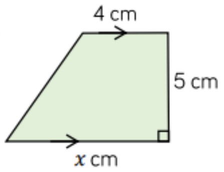
The steps are the same for solving and rearranging

Rearranging is often needed when using y = mx + c

eg Find the gradient of the line 2y − 4x = 9
Make y the subject first $y = \frac{4x + 9}{2}$ Gradient $= \frac{4}{2} = 2$

Form and solve equations and inequalities from shape

The area of the trapezium is $25cm^2$. Find x.



$$\frac{1}{2}(x + 4) \times 5 = 25cm^2$$
$$5(x + 4) = 50$$
$$5x + 20 = 50$$
$$5x = 30$$
$$x = 6cm$$

Change the subject of complex formula

$y = x^2 - a$
$x^2 = y + a$ — +a to both sides
$x = \pm\sqrt{y + a}$ — Don't forget the ± when you square root both sides

Factorise to isolate the 'x'
ax − cx = x(a − c)

$ax - b = cx + b$
$ax - cx = 2b$ — Rearrange so that all the terms involving 'x' are on the same side
$x(a - c) = 2b$
$x = \frac{2b}{a - c}$ — Divide by (a − c) to leave 'x' on it's own

Year 11 – Algebra...

Functions

HIGHER

What do I need to be able to do?

- Use function machines (R)
- Use function notation
- Work with composite functions (H)
- Work with inverse functions (H)

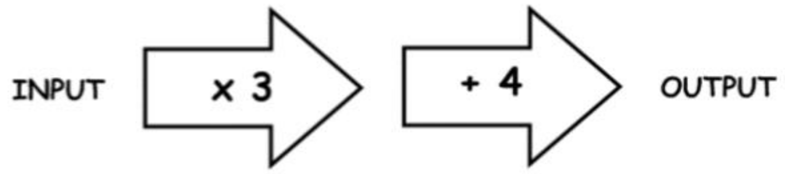
Keywords

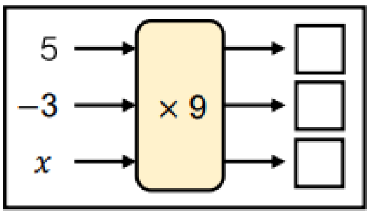
Input: a number/variable to put into an expression/function
Output: the result of a function
Function: a relationship between two sets of numbers – the input and output
Inverse: the opposite function
Variable: A symbol or letter for an unknown value
Composite: A function made of other functions. The output of one is the input of another
Rearrange: change the subject.

Function machines

R

Takes an **input** value, performs some **operations** and produces an **output** value.





Function notation

$$f(x) = 3x + 2$$

The function f is defined in terms of the variable x

To work out the value of f substitute the x value into $3x + 2$

e.g. $f(2) = 3(2) + 2$
$= 8$

The input value is 2 the output is 8

Functions don't always need to be $f(x)$ they can be given by any letter...

$$g(x) = 3x^2 - x - 7$$

$g(4) = 3 \times 4^2 - 4 - 7$
$= 3 \times 16 - 4 - 7$
$= 48 - 4 - 7$
$= 37$

Composite functions

A combination of two or more functions to create a new function

$fg(x)$ is the composite function that substitutes the function $g(x)$ into the function of $f(x)$

$fg(x)$... means 'do g first, then f
$gf(x)$... means 'do f first, then g

$$f(x) = 5x - 3 \qquad g(x) = \frac{1}{2}x + 1$$

a) What is $fg(4)$?

$g(4) = \frac{1}{2} \times 4 + 1$
$= 3$

$\text{f}(3) = 5 \times \mathbf{3} - 1$
$= 14$

Therefore: $fg(4) = 14$

b) What is $fg(x)$?

$fg(x) = 5\left(\frac{1}{2}x + 1\right) - 3$
$= \frac{5}{2}x + 5 - 3$
$fg(x) = \frac{5}{2}x + 2$

Inverse functions

$f^{-1}(x)$ - A function that performs the **opposite process** of the original function
"Really similar to changing the subject of formula"

Find $f^{-1}(x)$ given $f(x) = 3x + 4$

$y = 3x + 4$
$y - 4 = 3x$
$\frac{y-4}{3} = x$

$$\boldsymbol{f^{-1}(x) = \frac{x-4}{3}}$$

RULES FOR FINDING THE INVERSE $f^{-1}(x)$:

Step 1: Write out the function as $y = ...$

Step 2: Swap the x and y

Step 3: Make y the subject

Step 4: Instead of $y=$ write $f^{-1}(x) =$